

A SPACE $\times$ TIME FOR CONWAY’S GAME OF LIFE

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ABSTRACT. Conway’s Game of Life is a discrete model (discrete space and discrete time) of biological behavior based on blinking points on a lattice $\mathbb{Z}^2$. Although the Game of Life is clearly “geometric” in some sense (for example, one can refer to scenarios like ‘two gliders approaching each other’), it doesn’t seem very sensible to describe its discrete and dynamic geometry in terms of topological spaces.

Then, **what is the intrinsic geometry within Conway’s Game of Life?** While a topological space X can be regarded as a relative topos over (static) sets $\Gamma: \mathbf{Sh}(X) \rightarrow \mathbf{Set}$, we construct

a **relative topos** over the topos of discrete dynamical systems $\gamma: \sigma\text{-}\mathbf{PSh}(\mathbb{Z}^2, \text{Int}) \rightarrow \sigma\text{-}\mathbf{Set}$ and show that Conway’s Game of Life can naturally be regarded as an object of the topos $\sigma\text{-}\mathbf{PSh}(\mathbb{Z}^2, \text{Int})$.

The author must clarify that the mathematical contents are not very non-trivial so far.

Prepreprint v0.2 (20 September 2026). This work is in progress and does not constitute the complete version of the project.

This version combines the introductory motivation from an earlier manuscript with Sections 1–6 of the Advent note. The final remark in Section 6 and Section 7 (Remaining questions) of the Advent note are omitted.

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[Daw16, WHY ARE PEOPLE?]: Intelligent life on a planet comes of age when it first works out the reason for its own existence. If superior creatures from space ever visit earth, the first question they will ask, in order to assess the level of our civilization, is: ‘Have they discovered evolution yet?’

1. INTRODUCTION

1.1. Speculative Motivation. One of the ultimate goals of mathematics is to elucidate (the mathematical aspects of) the phenomena that exist in the real world. Indeed, mathematics has provided fundamental tools to diverse fields such as physics, economics, and public health. However, the author cannot help but feel that the most important phenomenon, **ourselves**, is being overlooked, though we as human beings would be explained by a (mathematical) structure, the principle of evolutionary theory¹.

This note is the first in a series of projects aimed at providing a mathematical formalization of evolutionary theory, particularly Dawkins' concept of the 'selfish gene' [Daw16]. Of course, countless mathematical models of evolutionary theory have been proposed.

Conceptual model (= structural model) vs Numerical model [Spi17]

Our goal is to construct a conceptual model (= structural model) for the principle of evolutionary theory, natural selection. More colloquially, our aim is to refer to the remarkable consequence of evolution, the existence of complex creatures (or "our existence"), from a structural perspective. More concretely, we seek to mathematically describe how complex individuals emerge solely from simple dynamical systems under the principle of natural selection.

By discussing evolutionary theory structurally (or more precisely, within the paradigm of modern mathematics), we aim to mathematically formalize the dynamical structure of evolutionary theory that leads to our existence in a materialistic world. The author cannot help but feel that this project is one of the most important themes in mathematics, a field that investigates the abstract structures of phenomena.

1.2. The implicit geometry (and logic). In tackling this highly popular yet simultaneously vague problem, it is important to clarify how our approach differs from previous ones. First and foremost, we confront the fundamental issue of the emergence of individuality, and for this reason, our approach to dynamical systems is inherently geometrical. We don't ignore the geometry implicit in natural selection.

The first step in the principles of evolutionary theory lies in the emergence of individuality. The emergence of individuals with temporal/spatial self-identity within a system that is merely given time evolution is far from trivial. (In fact, history has shown the danger of presupposing the concept of the individual. The errors in assuming individuality, such as organisms or species, have been pointed out in *The Selfish Gene*!)

The key concept is temporal/spatial self-identity. Time evolution, spatial extension, and the "logic imbued with ambiguous identity" that arises from them—I only know of one mathematical language that inherently possesses these qualities: **topos**². We will challenge the geometric and dynamical systems formulation of evolutionary theory by assigning a monoid operation, in this case, time action, to space.

1.3. First Step: Objective of this document. As the first step towards this grand speculative goal, we will begin by discussing the "individual." Since the structure we wish to discuss is the astonishing emergence of ourselves from a truly materialistic (flat and homogeneous) world, the existence of the "individual" should not be an axiom but the first theorem of our theory. In other words, we aim to derive the "individual" only from the assumption of a "time-evolving system."

The problem of the "emergence of individuals" is, of course, a topic with a long history of research and is by no means a novel idea. A well-known example is Conway's Life Game, which serves as an important toy example. In the Life Game, space is modeled by $\mathbb{Z}^2$ with local rules, and even though each cell merely blinks, we can refer to 'gliders moving to the upper right,' thus referring to "individuals."

¹Although this is widely recognized as a fact, it seems to be almost entirely misunderstood. Reducing our problems to evolutionary principles is not a simplistic analytical reduction, but rather something that becomes inevitable through continuous self-criticism.

²Note to my friend: You might think that I'm trying to apply topos theory to every random theory, here in particular, evolutionary theory. However, it's exactly the opposite: **Evolutionary theory is the reason why I chose topos theory.** To explain this fully, I would need to speak seriously about my materialist and skeptical beliefs as well as my geometric convictions, something that is beyond my current writing abilities.

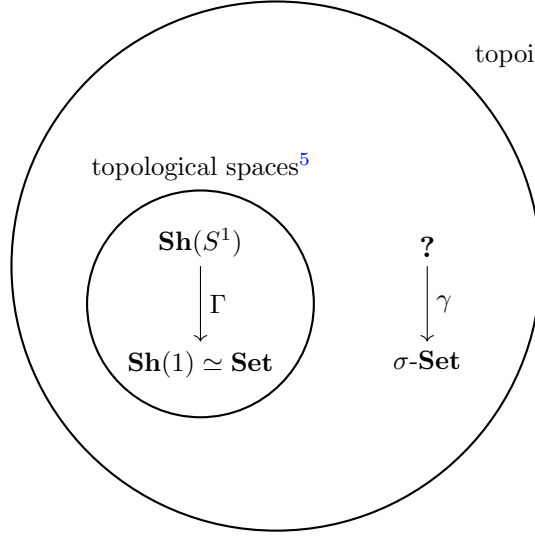


FIGURE 1. topoi subsume topology and dynamics

First, we will consider methods to describe such discrete-time/discrete-space systems.

1.4. Why topos? Conway’s Game of Life, which will be formally defined in definition 2.2, is a discrete model (discrete space and discrete time) of biological behavior (see fig. 2).

Our motivating question is:

Question 1.1. What is the intrinsic geometry within Conway’s Game of Life?

The first observation is that it has two completely different features compared to topological spaces:

Conway’s game of life is discrete: We consider a countable-size lattice $\mathbb{Z}^2$.

Conway’s game of life is dynamic: We consider a time-evolution of the state $\mathcal{P}(\mathbb{Z}^2) \rightarrow \mathcal{P}(\mathbb{Z}^2)$.

Hence, we’d better consider a broader framework of topology. Our framework to consider is not topological spaces, but **topos theory**, which is suitable to deal with discrete geometry.

Surprisingly, the topos-theoretic framework is not only suitable to deal with discrete space but also dynamical systems:

Definition 1.2 (The topos of discrete dynamical systems). A discrete dynamical system³ is a pair (X, f) of a set X and a function $X \xrightarrow{f} X$. The category of discrete dynamical systems (and the obvious notion of morphisms) is denoted by $\sigma\text{-Set}$ ⁴.

Since $\sigma\text{-Set}$ is just a presheaf category $\sigma\text{-Set} \simeq \mathbf{PSh}(\mathbb{N})$, it is a topos (See fig. 1).

Our question is now:

Question 1.3 (question 1.1, in terms of topoi). In what topos does Conway’s Game of Life live?

There are two trivial answers to question 1.3:

³it is also called a **difference set** in [Tom20]

⁴the notation is borrowed from [Tom20]

⁵It is more accurate to say locales.

Discrete: Conway’s game of life $(\{\top, \perp\}_{(x,y) \in \mathbb{Z}^2})$ is an object of the topos $\prod_{(x,y) \in \mathbb{Z}^2} \mathbf{Set}$

Dynamic: Conway’s game of life $\mathcal{P}(\mathbb{Z}^2) \rightarrow \mathcal{P}(\mathbb{Z}^2)$ is an object of the topos $\sigma\text{-}\mathbf{Set}$.

These naive answers are not satisfying since $\prod_{(x,y) \in \mathbb{Z}^2} \mathbf{Set}$ misses its dynamics, and $\sigma\text{-}\mathbf{Set}$ misses its geometry.

1.5. Why relative topos? This note aims to propose a better (not conclusive) answer to this question by considering a relative topos over the base topos $\sigma\text{-}\mathbf{Set}$.

Definition 1.4. A **relative topos** over the base topos $\mathcal{S}$, is a topos $\mathcal{E}$ equipped with a geometric morphism

$$\gamma: \mathcal{E} \rightarrow \mathcal{S}.$$

A Grothendieck topos is canonically a relative topos over the base topos $\mathbf{Set}$ by the global section geometric morphism $\Gamma: \mathcal{E} \rightarrow \mathbf{Set}$. For each object $X \in \mathcal{E}$, we have the **set** of global sections $\Gamma(X)$. Similarly, for a relative topos $\mathcal{E} \rightarrow \sigma\text{-}\mathbf{Set}$ over the base topos $\sigma\text{-}\mathbf{Set}$, each object $X \in \text{ob}(\mathcal{E})$ has the **discrete dynamical system** of global sections $\gamma_*(X) \in \text{ob}(\sigma\text{-}\mathbf{Set})$.

Therefore, the relative topos theory over $\sigma\text{-}\mathbf{Set}$ is a “dynamical geometry”, which is extensively studied in [Tom20]. With that knowledge, our final formulation of question 1.1 is the following:

Question 1.5. What is the suitable relative topos $\mathcal{E} \rightarrow \sigma\text{-}\mathbf{Set}$ such that

- there is an object **GoL**, called Conway’s game of life, and
- its global section $\gamma_*(\mathbf{GoL})$ coincides with the game of life map $\mathcal{P}(\mathbb{Z}^2) \rightarrow \mathcal{P}(\mathbb{Z}^2)$?

Proposition 6.4 provides an answer to this question, (but not in a conclusive way).

2. CONWAY’S GAME OF LIFE

Notation 2.1. We adopt the following notations.

- Let $\mathbf{2}$ denote the two-element (po)set of truth values $\mathbf{2} := \{0, 1\}$, which is interpreted as 0 = dead and 1 = alive.
- For a set X , let $\mathcal{P}(X)$ denote the set of functions $X \rightarrow \mathbf{2}$, which is canonically isomorphic to the powerset of X .
- For a point $(x, y) \in \mathbb{Z}^2$, its set of **neighbors** is $N_{(x,y)} := \{(x+i, y+j) \in \mathbb{Z}^2 \mid i, j \in \{-1, 0, 1\}\} \subset \mathbb{Z}^2$.

We define Conway’s game of life as an endofunction on $\mathcal{P}(\mathbb{Z}^2)$. If you don’t know the intuition behind it, see [Wikipedia], [YouTube: epic conway’s game of life], or something. There are lots of fascinating explanations online!

Definition 2.2. Conway’s game of life is the function $\text{con}: \mathcal{P}(\mathbb{Z}^2) \rightarrow \mathcal{P}(\mathbb{Z}^2)$ defined by

$$\text{con}(f)(x, y) := \begin{cases} 1 & \text{if } f(x, y) = 1 \text{ and } \#(N_{x,y} \cap f^{-1}(1)) \in \{3, 4\}, \\ 1 & \text{if } f(x, y) = 0 \text{ and } \#(N_{x,y} \cap f^{-1}(1)) = 3, \\ 0 & \text{otherwise.} \end{cases}$$

The glider is possibly the most famous phenomenon in Conway’s game of life (see fig. 2).

3. NAIVE OBSERVATIONS

We have simply defined Conway’s Game of Life as an endofunction, ignoring its geometric structure. So, what is the geometry here?

We want to capture two aspects:

- The rules are defined **locally**, and
- The whole state changes **globally**.

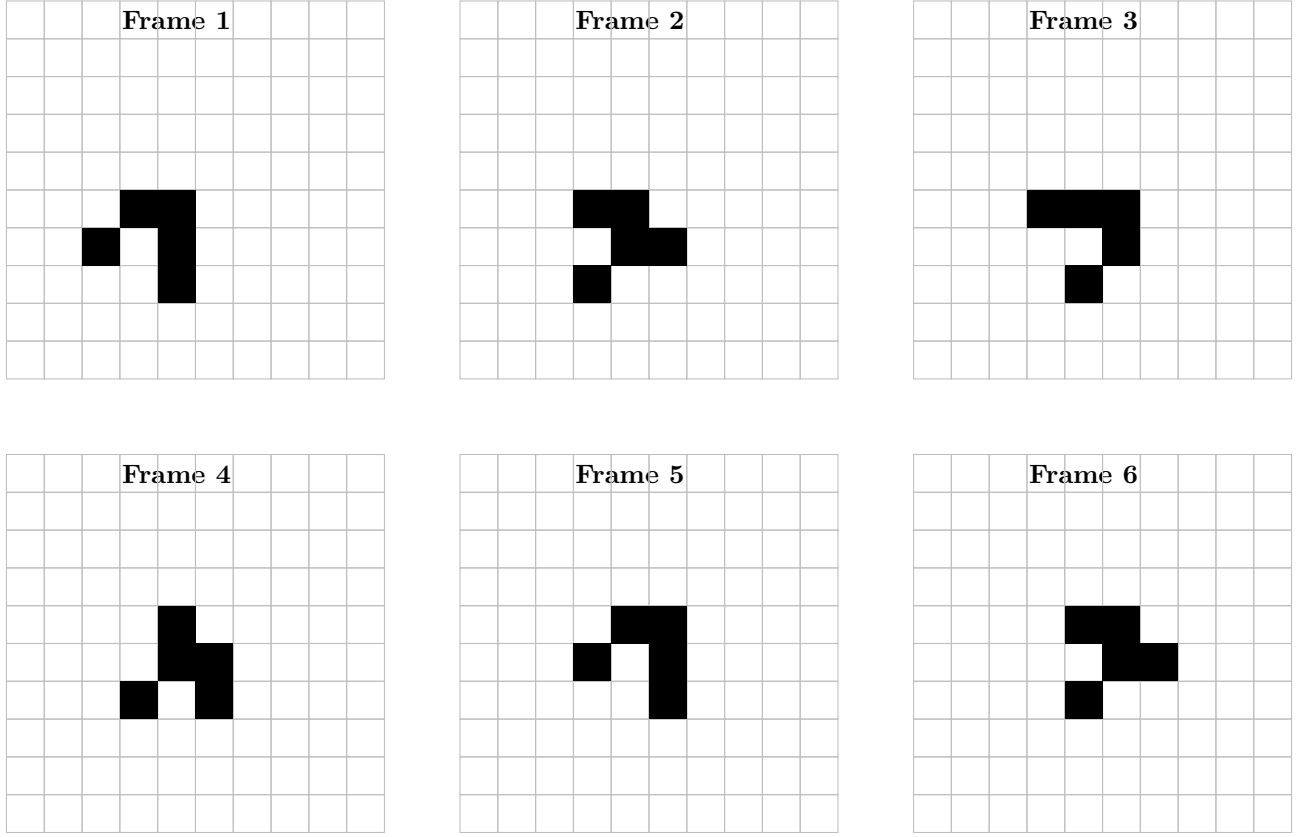


FIGURE 2. Moving glider

So a naive idea to capture the geometric aspect is to consider **sheaves** on a space $\mathbb{Z}^2$ valued in $\mathbf{2} := \{0, 1\}$

$$\mathbb{Z}^2 \supset U \rightarrow \mathbf{2}.$$

But there is an obvious problem: there is no suitable topology on $\mathbb{Z}^2$.

(For example, if $N_{(x,y)}$ is open for every $(x,y) \in \mathbb{Z}^2$, the space becomes discrete.)

So, we consider a generalized notion of interior operator. For a subset $S \subset \mathbb{Z}^2$ and a “current state” $S \rightarrow \mathbf{2}$, the next state

$$\text{Int}(S) \rightarrow \mathbf{2}$$

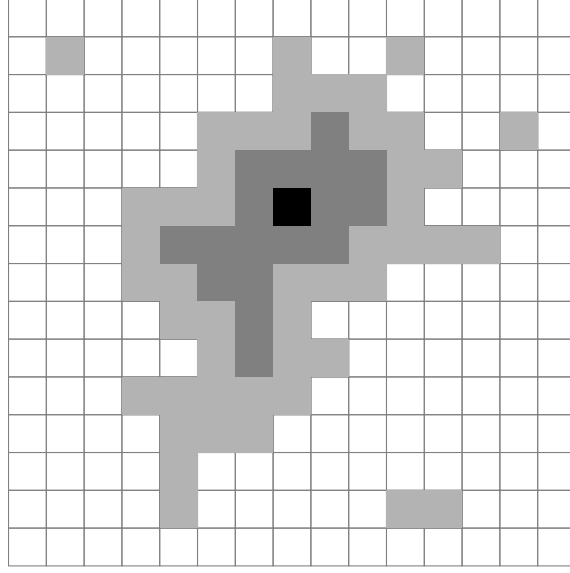
is defined on a smaller subset

$$\text{Int}(S) := \{(x, y) \in \mathbb{Z}^2 \mid N_{(x,y)} \subset S\} \subset S. \text{ (see fig. 3)}$$

So, we want to consider $\text{Int}: \mathcal{P}(\mathbb{Z}^2) \rightarrow \mathcal{P}(\mathbb{Z}^2)$ as an “interior operator” on the space $\mathbb{Z}^2$. The next section aims to formulate this idea.

4. PRETOPOLOGICAL SPACES

We start by recalling that topologies can be fully recovered by the corresponding interior operator:

FIGURE 3. $\text{Int}^2 \subsetneq \text{Int}$

Fact 4.1 (Topology in terms of interior operator). *Topologies on a set X are in one-to-one correspondence with interior operators⁶, i.e., a function $\text{Int}: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ such that*

lex: Int preserves finite inf. $\text{Int}(S \cap T) = \text{Int}(S) \cap \text{Int}(T)$ and $\text{Int}(X) = X$

counit: $\text{Int}(S) \subset S$

idempotent: $\text{Int}(S) = \text{Int}(\text{Int}(S))$

The condition “lex” implies that the function Int preserves the inclusion order $S \subset T \implies \text{Int}(S) \subset \text{Int}(T)$. We define pretopology as a dynamic version of topology, i.e., a non-idempotent interior operator.

Definition 4.2. A **pretopology**⁷ on a set X is a function $\text{Int}: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ such that

lex: Int preserves finite inf. $\text{Int}(S \cap T) = \text{Int}(S) \cap \text{Int}(T)$ and $\text{Int}(X) = X$

counit: $\text{Int}(S) \subset S$

A **pretopological space** is a set X equipped with a pretopology.

(This is a lex copointed endofunctor on $\mathcal{P}(X)$.)

Example 4.3. Every topological space is a pretopological space.

Example 4.4 (Graph). For a directed graph $(V, E \subset V^2)$, the function $\text{Int}: \mathcal{P}(V) \rightarrow \mathcal{P}(V)$ defined by

$$\text{Int}(S) := \{v \in S \mid v \rightarrow v' \implies v' \in S\}.$$

is a pretopology. If it is reflexive, i.e., $\{(v, v) \mid v \in V\} \subset E$, the graph structure is reconstructed by the induced pretopology, since $v \rightarrow v' \iff v \notin \text{Int}(V \setminus \{v'\})$.

Example 4.5. The pair $(\mathbb{Z}^2, \text{Int})$ in section 3 is a pretopological space induced by the following graph (Figure 4).

⁶categorically, this data is called a lex comonad on $\mathcal{P}(X)$.

⁷The same data, written in terms of closure operator, is called a Čech closure space. See [Rie21] for example.

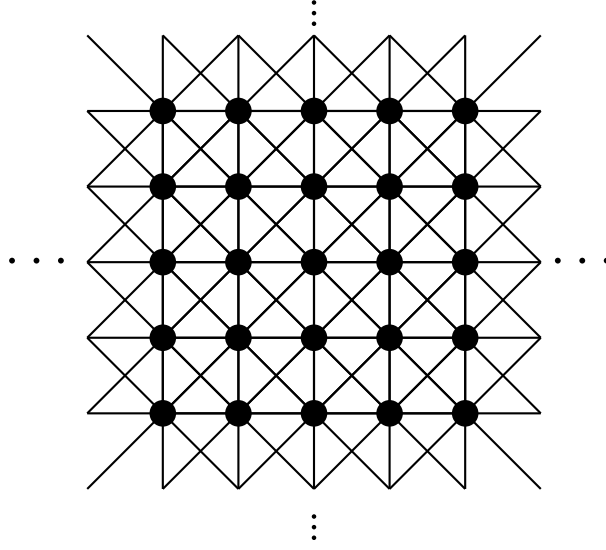


FIGURE 4. The graph on Conway's game of life

As a slogan, **pretopological spaces are a dynamical version of a topological space**, so that topological spaces coincide with “static” (i.e., $\text{IntInt} = \text{Int}$) pretopological spaces.

5. DYNAMICAL PRESHEAF TOPOS ON A PRETOPOLOGICAL SPACE

Definition 5.1. A **dynamical presheaf**⁸ on a pretopological space (X, Int) is a presheaf (= functor)

$$F: \mathcal{P}(X)^{\text{op}} \rightarrow \mathbf{Set}$$

equipped with a family of operators

$$\{\sigma_S: F(S) \rightarrow F(\text{Int}(S))\}_{S \in \mathcal{P}(X)}$$

such that

$$\begin{array}{ccc} F(S) & \xrightarrow{\sigma_S} & F(\text{Int}(S)) \\ \downarrow \text{res} & & \downarrow \text{res} \\ F(T) & \xrightarrow{\sigma_T} & F(\text{Int}(T)) \end{array}$$

commutes. The category of dynamical presheaves is denoted by $\sigma\text{-PSh}(X, \text{Int})$.

Example 5.2 (Game of life, as a dynamical presheaf). **Conway's game of life**, denoted by **GoL**, is a dynamical presheaf on the pretopological space $(\mathbb{Z}^2, \text{Int})$, defined to be a functor $\mathbf{GoL}: \mathcal{P}(\mathbb{Z}^2)^{\text{op}} \rightarrow \mathbf{Set}$

$$\mathbf{GoL}(S) := \mathcal{P}(S)$$

equipped with the restriction of Conway's game of life map $\text{con}: \mathcal{P}(S) \rightarrow \mathcal{P}(\text{Int}(S))$ (definition 2.2; cf. [FMS21, Definitions 18–19, Proposition 25]).

⁸This name might be confusing when the pretopological space is a topological space, since this does not coincide with the usual notion of presheaves.

Note that a pretopological space (X, Int) can be regarded as a functor

$$\mathbb{N} \rightarrow \mathbf{Cat}$$

that sends the unique object $*$ in $\text{ob}(\mathbb{N})$ to $\mathcal{P}(X) \in \text{ob}(\mathbf{Cat})$ and $1: * \rightarrow *$ to the interior operator $\text{Int}: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ ⁹.

Definition 5.3. For a pretopological space (X, Int)

we define the category $\mathcal{P}(X) \rtimes_{\text{Int}} \mathbb{N}$ to be the Grothendieck construction of the functor $\mathbb{N} \rightarrow \mathbf{Cat}$ described above.

More concretely, in the category $\mathcal{P}(X) \rtimes_{\text{Int}} \mathbb{N}$,

object: object is a subset of X

morphism: morphism from U to V is a natural number $n \in \mathbb{N}$ such that $U \subset \text{Int}^n(V)$

composition: the composite of $U \xrightarrow{n} V \xrightarrow{m} W$ is $U \xrightarrow{n+m} W$.

Proposition 5.4 (Dynamical presheaf is a presheaf). *For any pretopological space¹⁰, we have the following equivalence of categories:*

$$\sigma\text{-PSh}(X, \text{Int}) \simeq \text{PSh}(\mathcal{P}(X) \rtimes_{\text{Int}} \mathbb{N})$$

Proof. We define a category $C(X, \text{Int})$ by

objects: subsets of X ,

morphism $U \xrightarrow{\subset} V$: if $U \subset V$,

morphism $\text{Int}(U) \xrightarrow{1} U$: for any $U \in \mathcal{P}(X)$,

relations: $U_1 \xrightarrow{\subset} U_2 \xrightarrow{\subset} \dots \xrightarrow{\subset} U_n = U_1 \xrightarrow{\subset} U_n$ for any $n \geq 1$, and

relations: $\text{Int}(U) \xrightarrow{1} U \xrightarrow{\subset} V = \text{Int}(U) \xrightarrow{\subset} \text{Int}(V) \xrightarrow{1} V$ for any $U \subset V$.

Since $\text{PSh}(C(X, \text{Int})) \simeq \sigma\text{-PSh}(X, \text{Int})$, it is enough to prove $C(X, \text{Int}) \cong \mathcal{P}(X) \rtimes_{\text{Int}} \mathbb{N}$. Every morphism $U \rightarrow V$ can be shown to be equal to a morphism of the form of

$$U \xrightarrow{\subset} \text{Int}^n(V) \xrightarrow{1} \text{Int}^{n-1}(V) \xrightarrow{1} \dots \xrightarrow{1} \text{Int}(V) \xrightarrow{1} V.$$

This completes the proof. $\square$

Corollary 5.5. *The category $\sigma\text{-PSh}(\mathbb{Z}^2, \text{Int})$ is a presheaf topos, in which Conway's game of life **GoL** is an object.*

6. AS A RELATIVE TOPOS OVER THE TOPOS OF DISCRETE DYNAMICAL SYSTEMS

All of the content in this section is just an example of a general theory of relative topoi.

Proposition 6.1 (Special case of [Tom20, Section 13.13]). *A pretopological space (X, Int) defines an internal poset in the topos of discrete dynamical systems $\sigma\text{-Set}$.*

Proof. The object of objects is $\mathcal{P}(X)$ equipped with the endofunction Int . Since Int preserves the order, the object of morphisms $\{(U, V) \mid U \subset V\} \subset \mathcal{P}(X)^2$ is also an object of $\sigma\text{-Set}$. It is obvious that all the related maps, namely $\text{dom}, \text{cod}, \text{id}$, are morphisms in $\sigma\text{-Set}$, and they satisfy the equations for being an internal category. $\square$

Theorem 6.2 (Special case of [Tom20, Sections 13.14–13.16]). *A dynamical presheaf is precisely an internal presheaf of $\mathcal{P}(X)$ in the topos $\sigma\text{-Set} := \text{PSh}(\mathbb{N})$.*

$$\sigma\text{-PSh}(X, \text{Int}) \simeq \text{PSh}_{\sigma\text{-Set}}(\mathcal{P}(X), \text{Int})$$

In particular, the category of dynamical presheaves $\sigma\text{-PSh}(X)$ is a relative topos over the base topos $\sigma\text{-Set}$.

⁹Here, we only use the condition that Int preserves the inclusion order.

¹⁰Pretopology is too strong to prove this proposition. It is enough to assume that Int preserves the inclusion order.

Proof. Due to proposition 5.4, this is a special case of [Tom20, Sections 13.14–13.16]. The concrete calculation according to the definition of internal presheaves (see, for example, [MLM94, Chapter V, Section 7, pp. 242–244]) is not hard. $\square$

Lemma 6.3. *For a pretopological space (X, Int) , let $\pi: \mathcal{P}(X) \rtimes_{\text{Int}} \mathbb{N} \rightarrow \mathbb{N}$ be the projection functor, which sends $U \xrightarrow{n} V$ to $* \xrightarrow{n} *$. Then π admits both a left adjoint and a right adjoint.*

Proof. The fully faithful embedding $\iota_\emptyset: \mathbb{N} \rightarrow \mathcal{P}(X) \rtimes_{\text{Int}} \mathbb{N}: * \mapsto \emptyset$ is the left adjoint, and another fully faithful embedding $\iota_X: \mathbb{N} \rightarrow \mathcal{P}(X) \rtimes_{\text{Int}} \mathbb{N}: * \mapsto X$ is the right adjoint. $\square$

Proposition 6.4. *There is an adjoint quintuple*

$$\begin{array}{ccc}
 & \xleftarrow{\text{Lan}_{\iota_\emptyset}} & \\
 & \xrightarrow{\gamma_! = \text{ev}_\emptyset} & \\
 \sigma\text{-PSh}(X, \text{Int}) & \xleftarrow{\gamma^* = - \circ \pi} & \sigma\text{-Set.} \\
 & \xrightarrow{\gamma_* = \text{ev}_X} & \\
 & \xleftarrow{\text{Ran}_{\iota_X}} &
 \end{array}$$

Furthermore, the discrete dynamical system of global sections of **GoL**, which is defined to be $\gamma_*(\mathbf{GoL})$ is isomorphic to Conway’s game of life $\text{con}: \mathcal{P}(\mathbb{Z}^2) \rightarrow \mathcal{P}(\mathbb{Z}^2)$ (definition 2.2).

Proof. The existence of the adjoint quintuple is an immediate corollary of lemma 6.3. Since the global section functor γ_* is just the evaluation functor at $\mathbb{Z}^2$, the latter statement follows. $\square$

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