

$\square \quad \text{Hilb} \cong \text{Categorification}$

□ groupoid Cardinality

2 Möbius Inversion

③ Euler characteristics

($\frac{1}{n}$ summing z)

0 算数 e Categorification (From Finite sets to Feynman diagram)

(数論的構造の一般化 $\longleftrightarrow$ 代数構造)

小学校の代数 ... $\mathbb{N} = \{0, 1, 2, \dots\}$ a rig structure

たしざん -1

がつ	にち	なまえ
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たしざんを しましょう。



$$3 + 2 = \boxed{}$$



$$1 + 4 = \boxed{}$$



$$1 + 3 = \boxed{}$$



$$4 + 2 = \boxed{}$$



$$2 + 2 = \boxed{}$$



$$3 + 4 = \boxed{}$$



$$4 + 1 = \boxed{}$$



$$2 + 3 = \boxed{}$$

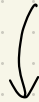
「 N is FinSet a categorification of 2 & 3 」

FinSet

$\{\circ \circ \circ\} \quad \{\circ \circ\}$



$\{\circ \circ \circ \circ \circ\}$



N

$$2+3=5$$

Rig-Category

Rig-Category とは、 $\mathcal{C}$ と $\mathcal{C}$ の

2つの monoidal structure

$\swarrow$ Symmetric
~~交換性~~

$$(\mathcal{C}, \otimes, 1_{\otimes}, \dots) \text{ と } (\mathcal{C}, \oplus, 0_{\oplus}, \dots)$$

$$\text{自然性: } C \otimes - : \mathcal{C} \longrightarrow \mathcal{C}$$

$\swarrow$ Naturality 3.1.

$\oplus$ の 構造を保つ

$\swarrow$ e.g.) strong monoidal morphism

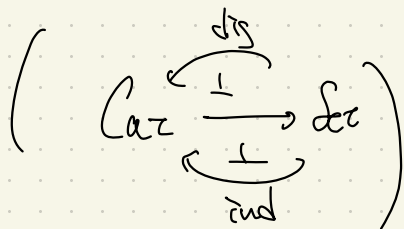
e.g.) Symmetric monoidal closed Category

(CCC, --- Fin, Set, Cat, ~~Poset~~, Heyting algebra H , ...
Vect_K,

自然性

$\text{Vect}(X)$: distributive monoidal category

e.g.) Rig R mod. R is on ind th $\text{Car} \Rightarrow \mathbb{Z} \Rightarrow \mathbb{Z} \Rightarrow \mathbb{Z}$



R a $\mathbb{Z} \in \mathbb{Z}$

monoidal structure
exists.

if $I_{\otimes} \cong O_{\otimes}$,

$$\begin{aligned} \text{then } \forall X \in \text{ob } \mathcal{C}, \quad X &\cong X \otimes I_{\otimes} \\ &\cong X \otimes O_{\otimes} \\ &\cong O_{\otimes} \end{aligned}$$

(ccc with zero object is 1-type)

Rig Category of $\mathbb{A}^1$ a function $\mathbb{Z}$ is not a $\mathbb{Z}$ structure.

e.g.) Deformation

a FinSec



同型数対射

a Sec

$\longrightarrow$ Cardinal Arithmetic.

($\leadsto$ 2nd order arithmetic ...)

$$d \quad \text{Fin}^{\mathbb{B}\mathbb{Z}} \longrightarrow \text{IL-}\gamma^a \text{ at } \mathbb{Q} \cdot (\mathbb{N}[x_1, x_2, \dots])$$

$$e \quad \text{Vect}_0^{\text{Fin BG}} \longrightarrow \textcircled{?} \quad \left(x_n \times x_m = \text{gcd}(n, m) \cdot x_{\text{lcm}(n, m)} \right)$$

$$\vdots$$

$$a \quad \text{Vect}_0 \longrightarrow \mathbb{N}$$

e.g) Categorical

$$\text{FinSet} \xrightarrow{\text{FinSet}_0} \mathbb{Q}[[x]]$$

$$\text{FinGroupoid} \xrightarrow[\text{Cardinality}]{\text{groupoid}} \mathbb{Q}_2$$

$$\text{Time Category} \longrightarrow \mathbb{R}_2$$

$$\text{Cat} \supset \textcircled{} \xrightarrow[\text{characteristics}]{\text{Euler}} \mathbb{Q}$$

1 Grouped Cardinality. (Base, $n=20$)

“国の濃度” は何 2 個ある?

	どの国 e.g.) ~~14~~ 4 国は いくつある?
国	label proper class
マシ	どの同型類の ~~個数~~ $2 \square$ $\frac{2}{4} \square, \frac{2}{2} \times \frac{2}{2} \square$
Grouped Cardinality	複数の同型も 含む “数え上げ” $\frac{2}{3} \square$

Def (Groupoid Cardinality)

$\mathcal{C}$: finite groupoid $\Rightarrow \mathbb{N}^2$.

$$|\mathcal{C}| := \sum_{[x] \in \pi_0(\mathcal{C})} \frac{1}{|\text{Aut}(x)|}$$



$\in \mathbb{R}^{\geq 0}$.

e.g.) finite set $X \Rightarrow \mathbb{N}^2$.

$|X|$ は X の元の数

e.g.)

$$\left| \begin{array}{c} \text{---} \curvearrowright \text{---} \\ \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \end{array} \right| = 1 + 1 + 1 = 3$$

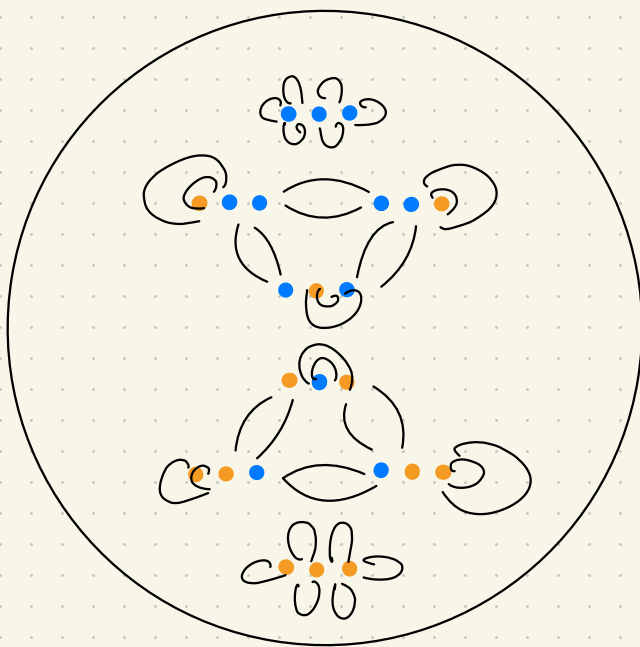
$$\left| \begin{array}{c} \text{---} \curvearrowright \text{---} \\ \bullet \quad \bullet \quad \bullet \quad \bullet \end{array} \right| = 1 + 1 + \frac{1}{2} = \frac{5}{2}$$

e.g.) G : finite group, X : finite set

$G \curvearrowright X$: 1 冫 冫 Δ'' $\lambda, 2, 1, 2$.

$T_G X$ --- translation groupoid $\hookrightarrow \mathcal{A}$.

$G_3 \curvearrowright \{ \bullet, \bullet \}^3$ (orbifold)



$$\frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \frac{1}{6} = \frac{4}{3}$$

$$|T_G X| = \frac{|X|}{|G|} \quad (\leadsto \text{orbit-stabilizer})$$

"local # 冫 冫" = "global # 冫 冫"

Remark (Categorification)

— principle of equivalence —

Groupoid^{fin}

$$\longrightarrow \mathbb{Q}_{\geq}$$

: Rig a Categorification

一般化. $\mathbb{Z} \subset \mathbb{Z} \subset \mathbb{Z}$. $\mathcal{C}$ a maximal groupoid $\mathcal{C}_0$
 $\Sigma \text{ (用)}$

$$|\mathcal{C}| := \sum_{[x] \in \pi_0(\mathcal{C})} \frac{1}{|\text{Aut}(x)|} \in \mathbb{R}_{\geq} \cup \{\infty\} \subset \widehat{\mathbb{R}}_{\geq}.$$

≠ 6" 424 43 5342 Σ tame category $\mathcal{C}$.)

$$\text{e.g.) } |\text{FinSet}| = \sum_{n \geq 0} \frac{1}{|S_n|} = e \quad \left(\begin{array}{l} \text{4242 / 1242120} \\ \text{Exp}(x) \end{array} \right)$$

$$|\text{FinVect}_{\mathbb{F}_p}| \quad (\hookrightarrow \mathbb{R}^{\geq 0} \text{ 224242})$$

$$|\text{FinSet}^{\rightarrow}| = e^e$$

⋮

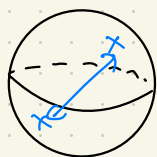
⋮
 Analytic Euler characteristic
 Combinatorics ...

e.g.) $X: \text{top.sp} \text{ (2012)}$

基本群の位数の逆数

$$|\pi_1(X)| = \sum_{c \in \pi_0(X)} \frac{1}{|\pi_1(c)|}$$

$\hookrightarrow$ Homotopy cardinality



S^2



$\mathbb{RP}^2$

$\sim$

1

$\frac{1}{2}$

$\cong 2$

"Universal Covering $\exists$ 何の n $\mathbb{Z}$ に $\mathbb{Z}^n$ に $\mathbb{Z}$ か?"

$(\mathbb{RP}^2 \text{ は } S^2 \text{ の "2倍"})$

$\rightsquigarrow$ Combinatorics (Total n species)

$\rightsquigarrow$ Higher & Enriched Category

$\rightsquigarrow$ Analytic approach

(π は k 上の正則関数に 1 の性質を証明する?)

例... $|e| = (\pi \text{ 関係の数})$

自然な e はある?

by Baez

2 Möbius inversion (整数 $\leadsto$ Poset (Poset theory))

Def (finite category の Möbius inversion)

finite category A 1222,

どゆ Q?
Ans 1. 逆行 (逆行) 2
Ans 2. 逆行 (逆行) 2
+ 分
+ 分

$$\sum_A : \mathbb{Q}^{\text{ob } A} \longrightarrow \mathbb{Q}^{\text{ob } A} : \mathbb{Q}\text{-linear}$$

$$\sum_A(a) = \sum_{b \in \text{ob } A} |A(a, b)| b$$

Category Algebra
= 代数

2 2222.

$\sum_A$ が 逆元 $\exists x \in A, \exists y \in A$
x 2222 μ_A 2222.

Remark (A , directed graph $\rightarrow$ 代数)

$\leadsto$ 代数. def は "finite", $\downarrow$ 2222.

A1, Category 22. 存在 $\uparrow$ 2222 2222.

A2, Euler 2222 $\exists x \in A, \exists y \in A$ 2222.

A	Σ_A	μ_A
$X: \text{finite set}$	$\begin{pmatrix} 1 & \cdots & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \cdots & 1 \end{pmatrix}$
$\mathbb{Q} \quad \bullet \longrightarrow \bullet$	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$
$\mathbb{N} \quad \bullet \rightarrow \bullet \rightarrow \cdots \bullet$	$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ & \ddots & \ddots & \vdots \\ 0 & & & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ 0 & & & -1 \end{pmatrix}$
$\mathbb{I} \quad \bullet \rightleftarrows \bullet$	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	存在せず principle of equiv. $\Rightarrow$ isobis?
$\text{Par} \quad \bullet \rightleftarrows \bullet$	$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$
$\triangle_{\leq n}^{\text{inj}}$ $\frac{1}{ \mathbb{A} } \mathbb{R} \quad n \text{元} \neq \text{ordinal}$ $\varepsilon, \text{order preserving}$ injective $\mathbb{I}$	$\begin{pmatrix} 0C_0, 1C_0, 2C_0, 3C_0 \\ & 1C_1, 2C_1, 3C_1 \\ & & 2C_2, 3C_2 \\ 0 & & & 1C_3 \end{pmatrix}$ $\mathbb{Q}[x] \leq n$ $n \quad x \mapsto x+1$ $x \mapsto x-1$ 0 1 2 3 とはいふ	$\begin{pmatrix} 0C_0, \ominus 1C_0, 2C_0, \ominus 3C_0 \\ & 1C_1, \ominus 2C_1, 3C_1 \\ & & 2C_2, \ominus 3C_2 \\ 0 & & & 1C_3 \end{pmatrix}$
$M: \text{finite monoid}$	$\begin{pmatrix} 1_M \end{pmatrix}$	$\begin{pmatrix} 1_M \end{pmatrix}$
$\vdots$	$\vdots$	$\vdots$

Rank (finite poset)

finite poset P 对 $\forall \mu_P \geq 0$.

再帰的に計算できる.

Rank (整数の χ の入関数との関係)

$|\text{ob} A| < \infty$ 对 $\mathbb{Q}^{\text{ob} A} \longrightarrow \mathbb{Q}^{\text{ob} A}$ と書けるが,

構成としては $\bigoplus_{a \in \text{ob} A} \mathbb{Q} a \longrightarrow \prod_{a \in \text{ob} A} \mathbb{Q} a$,

各 $b \in \text{ob} A \ni e_b$. $A(a, b) \neq \emptyset$ 对 $a \in \text{ob} A$ 对

有限個 (or locally finite) 对. 逆を考へる.

$(N \setminus \{0\}, |)$ は 条件をみたす.

計算対と整数 n 対 n 対 n 対 n .

$(\Delta^{\text{inv}} \neq)$

(the Noan of Robins invariant)
by Tamara over

Rank (Σ_A as categorified Yoneda (?))

Yoneda $\gamma_A : A \longrightarrow \text{Set}^{A^{\text{op}}}$

is $A(-, -) : A^{\text{op}} \times A \longrightarrow \text{Set}$ a Curry Ric ,
 $\downarrow$ $\downarrow$
 $a, b \longrightarrow A(a, b)$ (Cur = CCC)

$\mathbb{Q} \text{--} \mathbb{Q}$ $\Sigma_A : \mathbb{Q}^{ob A} \longrightarrow \mathbb{Q}^{ob A}$

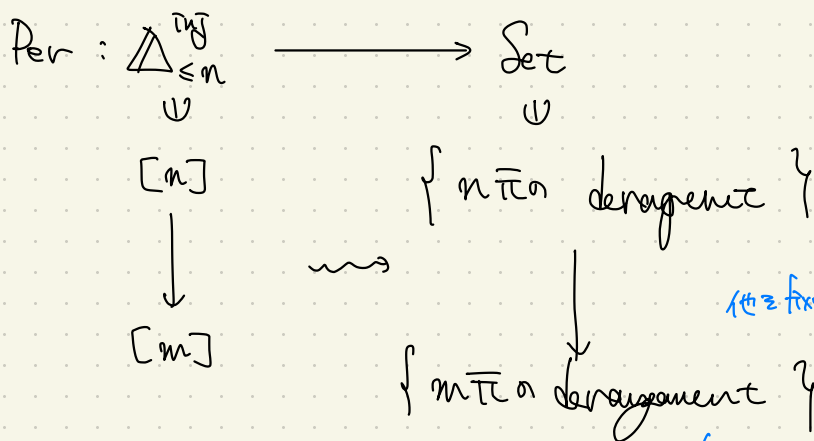
is $ob A \times ob A \longrightarrow \mathbb{N}$
 $\downarrow$ $\downarrow$
 $(a, b) \longmapsto |A(a, b)|$

is "symmetrization" $\mathbb{Q}^{ob A} \otimes \mathbb{Q}^{ob A} \longrightarrow \mathbb{Q}$ is Euler

(Vect $\mathbb{Q}$ is Symmetric monoidal closed structure)

e.g.) 組合せ論的群作用の例
(Rota's theory of $\in \leftarrow \pi$)

n 元集合の置換のうち, fixed point $\exists \neq i \mapsto i$ の
個数 d_n を調べる. derangement



1 は fixed point 123
 $\Rightarrow 1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 2$

$$\text{Per} \cong \sum_{0 \leq k \leq n} d_k \Delta_{\leq n}^{\text{inj}}(k, -)$$

$\Downarrow$ deconvolve

$$\text{Per}(*) = d_* \cdot \sum_{\Delta_{\leq n}^{\text{inj}}}$$

$\Downarrow$ Möbius inversion

$$\text{Per}(*) \cdot \mu_{\Delta_{\leq n}^{\text{inj}}} = d_*$$

$$\text{t.s.}, d_k = \sum_{0 \leq i \leq k} \underbrace{(i!)}_{| \text{Per}(i) |} \times \underbrace{(-1)^{i+k}}_{\mu_{\Delta_{\leq n}^{\text{inj}}}(i, k) = (-1)^{k-i}} \binom{k}{i} = k! \cdot \sum_{i=0}^k \frac{(-1)^i}{i!}$$

3 Euler characteristics

重みつき object の 個数.

(Notation set X is a $\mathbb{Z}$, $\mathbb{I}_X \in \mathbb{Q}^X$ is
the vector in $\mathbb{Q}^X$ pointing to X .)

Def (weighting)

finite category の weighting とは

$$k^\bullet \in \mathbb{Q}^{\text{ob}A} \text{ such that } \sum_A k^\bullet = \mathbb{I}_{\text{ob}A} \text{ holds.}$$

comweighting とは $k_\bullet \in \mathbb{Q}^{\text{ob}A} \text{ such that}$

$${}^t k_\bullet \sum_A = {}^t \mathbb{I}_{\text{ob}A} \text{ holds.}$$

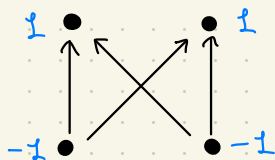
($\sum_{A \in \text{ob}A} k^\bullet_A = 1$, then "co" is not needed)

$$\forall a \in \text{ob}A \quad \sum_{b \in \text{ob}A} |A(a,b)| \cdot k^b = 1$$

"weighting of the number of objects"

e.g.) weighting

S^1



$\mathbb{I}$



$\mathbb{I}$

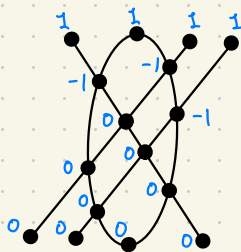


Uniqueness is $\mathbb{I}$

$\mathbb{I} \neq \mathbb{I}$ - $\mathbb{I}$

(finite poset

(with $\mathbb{I}$ and $\mathbb{I}$)



e.g.) co-weighting

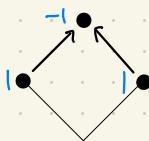
$\Lambda = \text{finite set}$ $\hookrightarrow \mathbb{N}^2$.

$P(\Lambda) \setminus \{\emptyset\}$

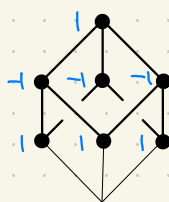
$P(0) \setminus \{\emptyset\}$



$P(2) \setminus \{\emptyset\}$



$P(3) \setminus \{\emptyset\}$



一般に, $S \in P(\Lambda) \setminus \{\emptyset\} \hookrightarrow \mathbb{N}^2$. $(-1)^{|S|+1}$ であるらしい。

$$0 = (1-1)^{|\Lambda|} = \sum_{i=0}^{|\Lambda|} (-1)^i \cdot |\Lambda| C_i$$

$$= 1 + \sum_{\substack{S' \subseteq S \\ S' \neq \emptyset}} (-1)^{|S'|}$$

$$\text{よって} \sum_{\substack{S' \subseteq S \\ S' \neq \emptyset}} (-1)^{|S'|+1} = 1 \quad \text{---}$$

Lemma (weighting on 成り分和)

$\Lambda = \text{finite category}$ $\hookrightarrow \mathbb{N}^2$. Λ 上の weighting $k^* \in \text{co-weighting } k$.
 $\in \text{成り分和}$, k^* の成り分和 $\in k$ の成り分和は 等しい。

(proof)

$${}^t k \cdot \mathbb{I}_{\text{cat}} = {}^t k \cdot \sum_{\Lambda} k^* = {}^t \mathbb{I}_{\text{cat}} \cdot k$$

Def (Euler characteristic)

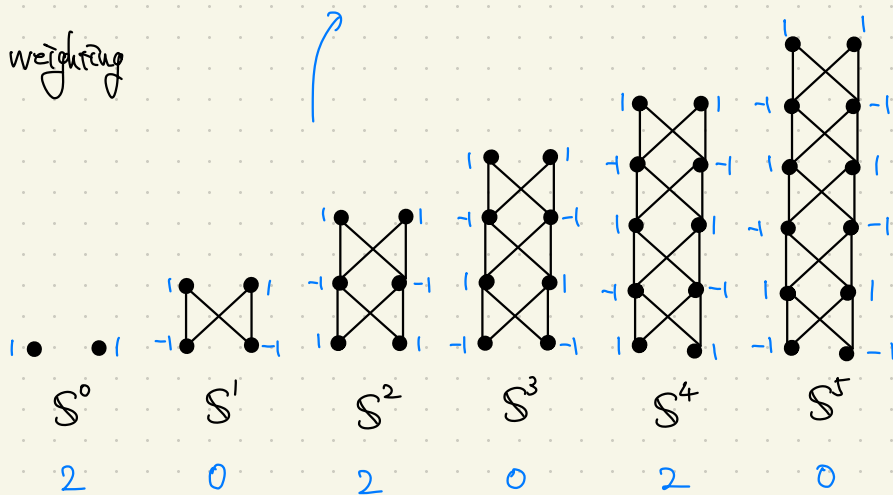
finite category $\mathcal{A}$ w/ weighting $\neq$ coweighting $\neq$

$$\in \mathbb{Q} \neq \mathbb{Z}, \quad \chi(\mathcal{A}) := \sum_{a \in \text{ob } \mathcal{A}} k^a = \sum_{a \in \text{ob } \mathcal{A}} k_a \quad (\in \mathbb{Q})$$

$\in \frac{1}{N} \mathbb{Z}$.

Unique rational $\neq \mathbb{Z}$ & "ob" ...

(Remark (Categorification of $\mathbb{Q}$)
Cat With Euler Characteristic $\longrightarrow \mathbb{Q}$
Rig a Categorifier
(classifying space w/ $S^1, S^2, \dots$)



Prop (χ is Groupoid Cardinality)

A : finite groupoid th.

$$|A| = \chi(A)$$

(proof)

A : connected $\in \mathbb{Z} \neq 4$

$a \in \text{ob } A$ の 射の 個数は $|\text{Aut}(a)| \times |\text{ob}(A)|$

に等しい。各 object a weightage $\geq \frac{1}{|\text{ob}(A)| \times |\text{Aut}(a)|}$

と 確かでない、

Prop (χ is Möbius inversion)

A : finite category is a 2.

μ_A exists $\iff$ weighting is unique

$\iff$ cweighting is unique

($\Rightarrow$ Euler 数 $\chi(A)$ exists) is a 2.

is a 2. this, $\chi(A) = (\mu_A \text{ of sum})$

(proof)

μ_A exists $\iff \zeta_A$ is invertible

$\iff \zeta_A \cdot k \neq 0 \ \forall k \in \mathbb{Q}^{\text{ob}A}$ is unique

$\iff {}^t k \cdot \zeta_A \neq 0 \ \forall k \in \mathbb{Q}^{\text{ob}A}$ is unique

$$\chi(A) = {}^t \mathbb{I}_{\text{ob}A} \cdot k = {}^t \mathbb{I}_{\text{ob}A} \cdot \mu_A \cdot \zeta_A \cdot k$$

$$= {}^t \mathbb{I}_{\text{ob}A} \mu_A \cdot \mathbb{I}_{\text{ob}A}$$

A	$\chi(A)$	μ_A
X : finite set	$ X $	$\begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix}$
$\mathbb{D} \quad \bullet \longrightarrow \bullet$	1 <i>initial 31)</i>	$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$
$\mathbb{N} \quad \bullet \rightarrow \bullet \rightarrow \dots \bullet$	1 <i>initial 31)</i>	$\begin{pmatrix} 1 & -1 & & \\ & \ddots & \ddots & \\ 0 & & \ddots & -1 \\ & & & 1 \end{pmatrix}$
$\mathbb{I} \quad \bullet \rightleftarrows \bullet$	1 <i>initial 31)</i>	存在せず
$\text{Par} \quad \bullet \rightleftarrows \bullet$	0	$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$
$\Delta_{\leq n}^{\text{inj}}$ $\frac{1}{n!} \mathbb{R}$ n 元 $\neq$ ordinal ε , order preserving injective $\boxed{\text{真}}$	1 <i>initial 31)</i>	$\begin{pmatrix} \overset{1}{\textcircled{0}C_0}, \overset{0}{\textcircled{-1}C_0}, \overset{0}{\textcircled{2}C_0}, \overset{0}{\textcircled{-3}C_0} \\ \textcircled{C_1}, \textcircled{-2C_1}, \textcircled{3C_1} \\ 0, \textcircled{2C_2}, \textcircled{-3C_2} \\ \delta_{0,n} = (1-1)^n, \textcircled{3C_3}, \end{pmatrix}$
M : finite monoid	$\frac{1}{ M }$	$\left(\frac{1}{ M } \right)$

Prop (adjunction の類ごとの不変量)

有限圏 A, B について.

(i) $A \simeq B$ (equiv of cat) ならば

$A \simeq B$ ならば 1-射の存在は同値

その値も等しい.

これは h の不変量,
typical なのは 連結成分

(ii) $A \xrightleftharpoons[G]{F} B$: adjunction ならば

$A \simeq B$ ならば 1-射 3 つも存在.

$$\chi(A) = \chi(B)$$

(sketch of proof)

(i) 同型類の個数だけ重みを変えるだけ

(ii) 関手 $G: A \longrightarrow B$ での object a について

$$\nu(G) = \mathbb{Q}^{ob A} \longrightarrow \mathbb{Q}^{ob B} \text{ 定義可}$$

$$F + G \text{ により } {}^t \nu(F) \zeta_A = \zeta_B \nu(G) \text{ となる,}$$

$$\chi(A) = \underbrace{{}^t k_B \cdot \zeta_B}_{\underbrace{{}^t I_{ob B}}_{\underbrace{{}^t I_{ob A}}}} \cdot \nu(G) \cdot \underbrace{k_A}_{\underbrace{I_{ob A}}} = \underbrace{{}^t k_B \cdot {}^t \nu(F) \cdot \zeta_A}_{\underbrace{{}^t I_{ob B}}} \cdot k_A = \chi(B)$$

その文章では 3つの 整合性

- Top.sp. a 3k
- Orbifold a 3k ($\mathbb{Q}$ -valued Euler characteristic)
- グラフ a 3k

その一致をみたす。

一番簡単なグラフで示す。

$F(G)$: G の
自由多面体
が future である。

Prop (グラフの Euler 数, "v-e")

$G = (E, V)$: サイクルを含まない 有向グラフ

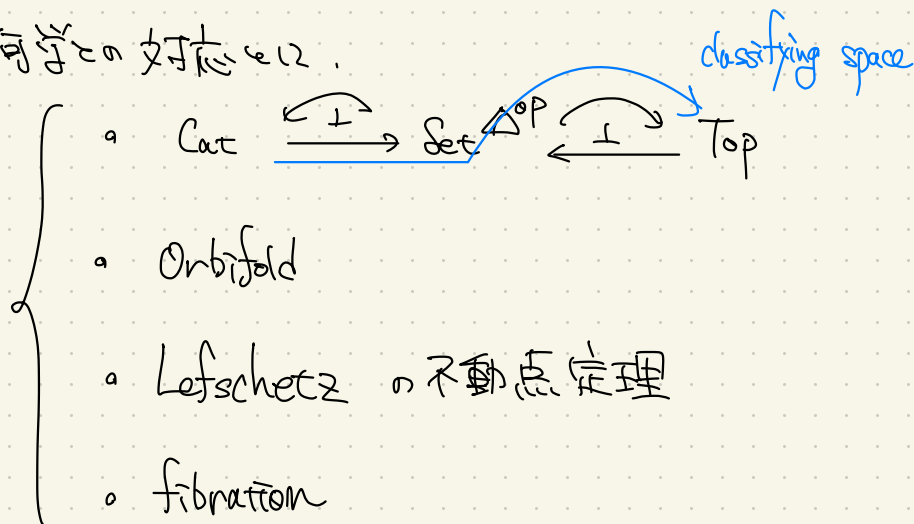
つまり, $\chi(F(G)) = |V| - |E|$

(proof) $\zeta_G: \mathbb{Q}^V \longrightarrow \mathbb{Q}^V$ は 図のときと同様に
定義する。このとき, $\zeta_{F(G)} = \sum_{n=0}^{\infty} \zeta_G^n$ 実質有限和と表す。

$$\zeta_{F(G)} = I_V + \zeta_G \cdot \zeta_{F(G)} \quad \text{or} \quad \mu_{F(G)} = I_V - \zeta_G$$

$\mu_{F(G)}$ の成分和は $|V| - |E|$

幾何学との対応を.



が挙げられている。

最後のを

discrete fibration のときだけ示す。

$$F: \mathcal{C} \longrightarrow \text{FinSet} \left(\longleftrightarrow \text{Set} \right)$$

1-2.2.

$$\begin{array}{ccc}
 \int F & \xrightarrow{\quad} & \text{FinSet}_* \\
 \downarrow & \lrcorner & \downarrow \\
 \mathcal{C} & \xrightarrow{F} & \text{FinSet}
 \end{array}$$

category of elements は

必要なのは (Path-lifting property) 2 特徴 3

Prop (Euler characteristic of $\int F$)

A : finite category , k : A -weighting

$F : A \longrightarrow \mathbf{FinSet}$: functor $\exists$,

$\int F$ の object (a, x) ($a \in \text{ob} A$, $x \in Fa$)

$\exists k^{(a,x)} := k^a \in \mathbb{Z}$,

$k^{(a,x)}$ は weighting である。 $\in \mathbb{Z}$,

$\int F$ の 1つの 数 $\in \mathbb{Z}$ (e.g. A : $\mathbb{Z} \uparrow \text{poset}$)

$$\chi(\int F) = \sum_{a \in \text{ob} A} |Fa| \times k^a \in \mathbb{Z} .$$

(sketch of proof)

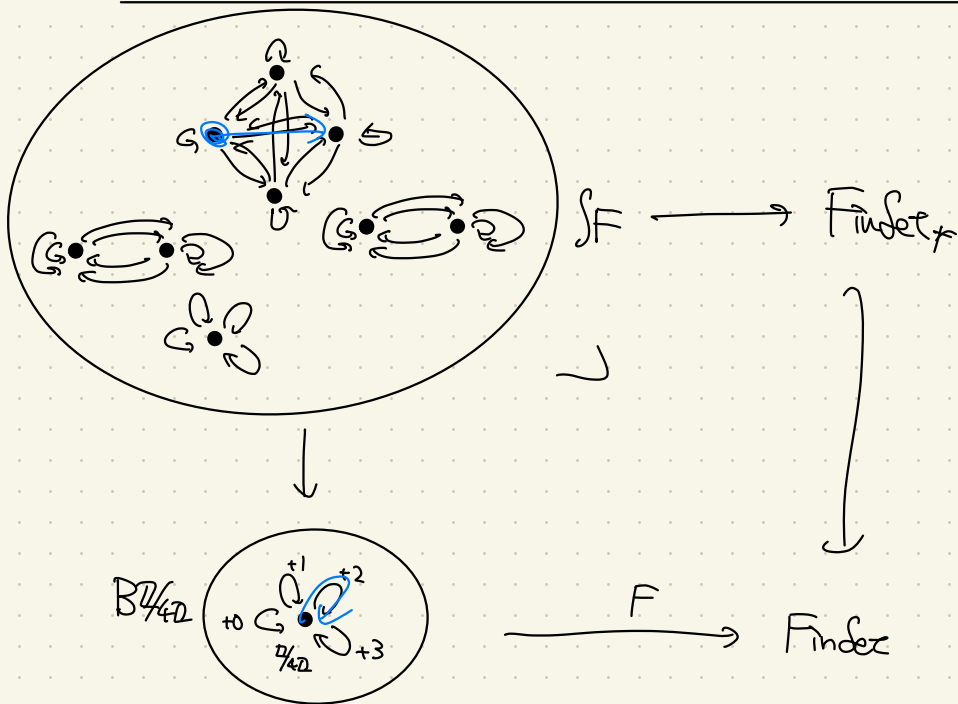
a 1つの 数 (a, x) の 1つの Unique に 持てた。

e.g.) Translation groupoid a groupoid cardinality

$G \curvearrowright X$: 有限群 G の有限集合 X の作用

$$\text{if } BG \xrightarrow{F} \text{FinSet} \text{ is a 2-functor,}$$

$$|T_G X| = |X| \times \frac{1}{|G|} \quad \text{e.g. } \frac{1}{24}$$



e.g.) 包除原理

$\{A_\lambda\}_{\lambda \in \Lambda}$: 有限集合の有限族

$$F: (P(\Lambda) \setminus \{\emptyset\})^{\text{op}} \longrightarrow \text{FinSet}$$

$$\begin{array}{ccc} \psi & & \psi \\ S & \longmapsto & \bigcap_{\lambda \in S} A_\lambda \end{array}$$

$$\chi(S_F) = \left| \bigcup_{\lambda \in \Lambda} A_\lambda \right| \quad \left(\begin{array}{l} \text{Category of elements} \\ \text{の各連結成分は} \\ \bigcup_{\lambda \in \Lambda} A_\lambda \text{ の元は互いに} \\ \text{互いに initial である} \end{array} \right)$$

$$\# \left| \bigcup_{\lambda \in \Lambda} A_\lambda \right| = \sum_{\substack{S \subseteq \Lambda \\ S \neq \emptyset}} (-1)^{|S|+1} \cdot \left| \bigcap_{\lambda \in S} A_\lambda \right|$$

$\hookrightarrow$ Cauchy completeness を用いると良い
説明は後述

4!

• Categorification of analytic combinatorics -

• The Continuity of combinatorics

• $\times$ どんな図が (半環 $\mathbb{Q}$ に) Euler 数をもつか?
• $\circ$ Euler 数 0 値をもつべき代数は何か?

• Enriched ver

Finitary-enriched $\mathbb{Q}$ object $X := |X| \in \mathbb{Q}$
 $\mathbb{Q} \ni \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

$\text{Vect}_{\mathbb{Q}}^{\text{fin}}$ #3 次元をみる

• Higher (高次元 $\mathbb{Q}$ -代数)

⋮